EE4902 Part 2 Assignment 1

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# Is Gaussian smoothing operator a linear or non-linear filter? Explain your answer and illustrate it with images.

The Gaussian smoothing operator is a linear filter. The principle of superposition holds. i.e. if input signal is , then the output signal is for any constants and .

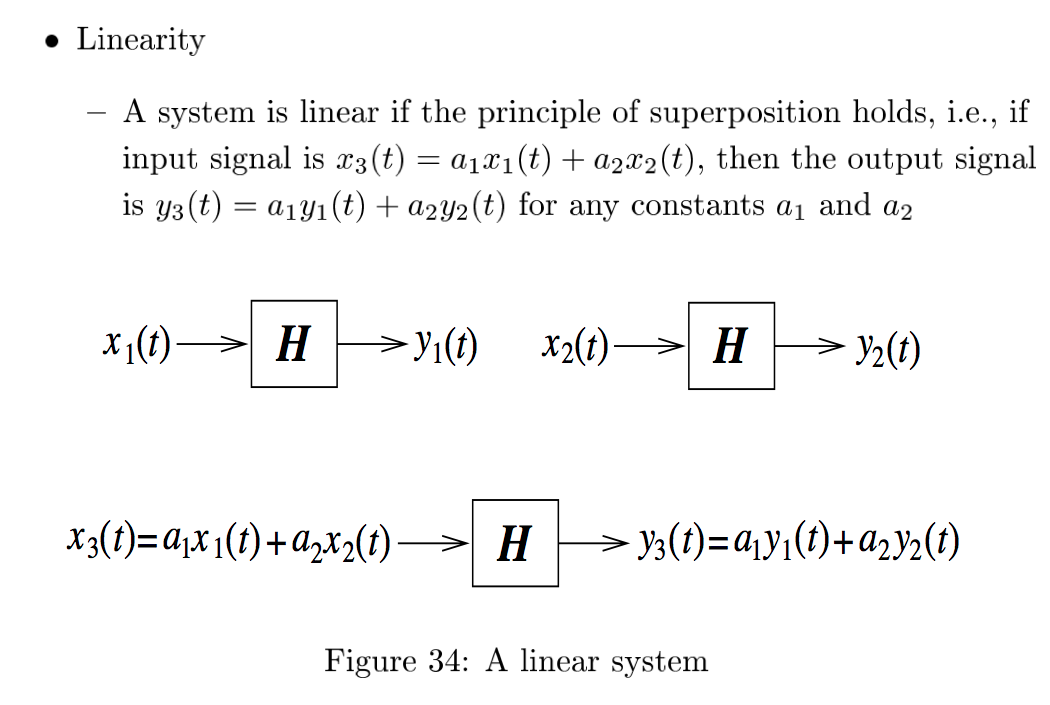


Figure ‑ Linear transformation

With respect to Figure 1‑1, is the original image, while is the linear scaling coefficient that is applied to the image to obtain the contrast stretched image. is the Gaussian blur. When x is transformed by H, it yields y, a Gaussian blurred image.

To illustrate it with images, Gaussian blurring two images and then summing them up will produce the same image as summing two image and then Gaussian blurring them. In the following images, a Gaussian blur of size=15, and sigma = 12 was applied.

|  |  |  |
| --- | --- | --- |
| Normalized Image 1 | Normalized Image 2 | Sum of both images and Gaussian blurred |
|  |  |  |

|  |  |  |
| --- | --- | --- |
| Normalized and Gaussian Blurred Image 1 | Normalized and Gaussian Blurred Image 2 | Sum of both images |
|  |  |  |

The result is equal. This was showed by subtracting the resulting images from each other. What was obtained was an image with all zeroes as its pixel values, indicating that the two images were equal.



# Is median filter a linear or non-linear filter? Show that it is or is not and illustrate it with images.

A median filter is a non-linear filter. It replacing the middle pixel by the median pixel in the neighborhood and is hence unaffected by linear scaling of the image pixels. This makes it particularly suitable for impulse noise. If the median filter were linear, applying a linear transformation to the image with the noise will result in a linear transformation to the noise pixels. However, the results show that applying a linear scaling onto the original image has little to no effect of the removal of the impulse noise.

Salt and pepper noise was applied to three different images: (1) the original image 92) a contrast stretched image and (3) a Gaussian blurred image. Whether the image is scaled or blurred, the median filter can remove the salt and pepper noise just as well.

|  |  |  |  |
| --- | --- | --- | --- |
| Normalized Image 1 with impulse noise | Normalized Image 2 with impulse noise | Sum of both | Median Filter |
|  |  |  |  |

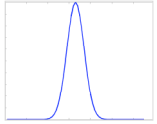
|  |  |  |  |
| --- | --- | --- | --- |
| Normalized Image 1 | Normalized Image 2 | Salt and pepper noise applied to sum of both | Median Filter |
|  |  |  |  |

The results are not equal. This was showed by subtracting the resulting images from each other. What was obtained was an image with mostly zeroes as its pixel values, but there are some spots in the resulting image, showing that the images are not equal.



# Explain how the Laplacian of Gaussian (LoG) operator can be implemented by 1D filters?

The LoG is implemented by first convoluting the original image with a Gaussian kernel.



Afterwards, a Laplacian operator is performed on the Gaussian smoothed image.

The first step to doing this is to convolute the image with a 1-D first derivative filter in the x direction. This will obtain the zero crossing.

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
| Direction of Convolution  🡪 | \* | |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | 66 | 65 | 62 | 63 | 64 | 62 | | 65 | 63 | 61 | 60 | 61 | 62 | | 55 | 60 | 58 | 57 | 56 | 55 | | 56 | 58 | 59 | 56 | 57 | 58 | | 57 | 60 | 61 | 60 | 57 | 56 | | 59 | 55 | 54 | 56 | 56 | 57 | |  |
| |  |  |  | | --- | --- | --- | | -1 | 2 | -1 | |

After the Laplacian is performed, the zero crossing is found. Performing a second derivative of a Gaussian in the y direction will yield a Mexican hat operator, which is the Laplacian of Gaussian.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Direction of convolution  ↓ | |  | | --- | | -1 | | 2 | | -1 | |  |
| |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | 66 | 65 | 62 | 63 | 64 | 62 | | 65 | 63 | 61 | 60 | 61 | 62 | | 55 | 60 | 58 | 57 | 56 | 55 | | 56 | 58 | 59 | 56 | 57 | 58 | | 57 | 60 | 61 | 60 | 57 | 56 | | 59 | 55 | 54 | 56 | 56 | 57 | | |  |

It is possible to generate a LoG using two Gaussian filters because of the property of the separability property of the Gaussian. This results in significant computation cost savings as well.